It has been suggested the overall probability of n events each with probability p_i is:

$$P = 1 - \sqrt[n]{\prod_{i=1}^{n}(1 - p_i)} \tag{1}$$

This is clearly not true in the case of 3 events, each with the same probability p where it would be expected the overall probability is p^3

$$P = 1 - \sqrt[3]{(1-p)^3} = 1 - (1 - p) = p \tag{2}$$

Probability is defined as, providing all events are equally likely:

$$\frac{number\ of\ occurrences\ of\ an\ event}{sample\ space\ (set\ of\ all\ possible\ outcomes)} \tag{3}$$

A 6 sided die with equal probability of landing on each side and with sides labelled 1,2,3,4,5,6

So the probability of throwing a 1 is $\frac{1}{6}$ and the probability of not throwing a 1 is $1 - \frac{1}{6} = \frac{5}{6}$

The sample space for 2 throws has 36 possible events

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

The probability of throwing 2 ones is $(\frac{1}{6})^2 = \frac{1}{36}$ which corresponds to the cell coloured yellow in the table above.

So the probability of not throwing 2 ones is $\frac{35}{36}$

But trying to calculate this as $1 - (\textit{probability of not throwing a } 1)^2$ (note this is just equation 1 without the square root).

$$1 - (\frac{5}{6})^2 = 1 - \frac{25}{36} = \frac{11}{36} \tag{4}$$

which is wrong, it is actually the probability of throwing at least 1 one, which can be seen by counting the cells coloured yellow and green in the table above.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Equation 1 gives

$$P = 1 - \sqrt[2]{(1 - \frac{1}{6})^2} = 1 - (1 - \frac{1}{6}) = \frac{1}{6} \tag{5}$$

which is clearly wrong

But $(1-(probability\ of\ not\ throwing\ a\ 1))^2$ gives the right answer

We can calculate as

$$(1-\frac{5}{6})^2 = 1-\frac{10}{6}+\frac{25}{36} = \frac{36-60+25}{36} = \frac{1}{36} \tag{6}$$

or more simply.

$$(1-\frac{5}{6})^2 = (\frac{1}{6})^2 = \frac{1}{36} \tag{7}$$

Notes on probability

Assume a sameple space S

for each event E, of the sample space S, we assume that P(E) is defined which satisfies the following 3 conditions

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. for any sequence of events $E_1, E_2, ...$ which are mutually exclusive, that is $E_m E_n = \emptyset$ where $m \neq n$

$$P(\bigcup_{n=1}^{\infty} E_n) = \sum_{n=1}^{\infty} P(E_n) \tag{8}$$

We refer to $P(E)$ as the probability of the event E. E^c is called the compliment of E and consists of all events in the sample space which are not in E. Thus

1. $E \cup E^c = S$
2. $P(E) + P(E^c) = 1$

Events E and F are said to be independent if

$$P(EF) = P(E)P(F) \tag{9}$$

which implies

$$P(E|F) = P(E) \tag{10}$$

An example of independent events (assumed independent) are throwing a dice twice and getting 1 followed by 2 $P(1)P(2) = \frac{1}{6} \times \frac{1}{6}$.

For events E and F they may contain some of the same points in the sample space $(E \cup F \neq \emptyset)$. Thus

$$P(E) + P(F) = P(E \cup F) + P(EF) \tag{11}$$

Alternatively

$$P(E \cup F) = P(E) + P(F) - P(EF) \tag{12}$$

When E and F are mutually exclusive

$$P(E \cup F) = P(E) + P(F) - P(\emptyset) \tag{13}$$
$$= P(E) + P(F) \tag{14}$$

An example of mutually exclusive events are

1. $P(E)$ = probability of getting an even number less than 4 on throwing a dice $(\frac{1}{6})$
2. $P(F)$ = probability of getting an odd number less than 4 on throwing a dice $(\frac{2}{6})$

Note

1. $P(E) + P(F) = \frac{3}{6}$
2. $P(\textit{a number less than } 4 \textit{ on a dice throw} = \frac{3}{6})$